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Hawking–Unruh Effect on Thermal Equilibrium State

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It is pointed out that the usual (Gibbs) thermal equilibrium state in Minkowski spacetime is no longer a thermal equilibrium state for a uniformly accelerated observer. Similarly, the thermal equilibrium state in Kruskal spacetime is not a thermal state for a Schwarzschild observer.

It is well known that a vacuum state in Minkowski spacetime appears to be a thermal state for a uniformly accelerated Rindler observer (Sciama, 1981; Gibbons and Perry, 1978). Unruh gives a simple proof (Unruh, 1976; Birrell and Davis, 1982). Under the Rindler transformation

$$\begin{cases} t = a^{-1} e^{a\xi} \operatorname{sh}(a\eta) \\ x = a^{-1} e^{a\xi} \operatorname{ch}(a\eta) \end{cases} \tag{1}$$

the line element is represented as

$$ds^2 = e^{2a\xi}(d\eta^2 - d\xi^2) \tag{2}$$

where $ae^{-a\xi}$ is the proper acceleration of the Rindler observer at ξ. For a massless scalar field, Unruh obtains the Bogolubov transformations

$$\begin{cases} b_K^{(1)} = [2 \operatorname{sh}(\pi\omega/a)]^{-1/2}[e^{\pi\omega/2a} d_K^{(2)} + e^{-\pi\omega/2a} d_{-K}^{(1)\dagger}] \\ b_K^{(2)} = [2 \operatorname{sh}(\pi\omega/a)]^{-1/2}[e^{\pi\omega/2a} d_K^{(1)} + e^{-\pi\omega/2a} d_{-K}^{(2)\dagger}] \end{cases} \tag{3}$$

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which provide a relation between a Minkowski vacuum state $|0\rangle_M$ and a Rindler vacuum state $|0\rangle_R$, where

$$d_K^{(1)}|0\rangle_M = d_K^{(2)}|0\rangle_M = 0 \qquad (4)$$
$$b_K^{(1)}|0\rangle_R = b_K^{(2)}|0\rangle_R = 0$$

and $\omega = |K|$. In fact, $b_K^{(1)}$ and $b_K^{(2)}$ are annihilation operators of the Rindler particles in the Rindler regions L and R, respectively (see Fig. 1). The Hermitian adjoint operators $b_K^{(1)\dagger}$ and $b_K^{(2)\dagger}$ are the respective creation operators of these particles. On the other hand, both $d_K^{(1)}$ and $d_K^{(2)}$ are annihilation operators of the Minkowski particles almost entirely concentrated in the Rindler regions L and R, respectively. And both $d_K^{(1)\dagger}$ and $d_K^{(2)\dagger}$ are creation operators of them (Unruh, 1976).

From (3), we have

$$b_K^{(1)\dagger} = [2\,\mathrm{sh}(\pi\omega/a)]^{-1/2}[e^{\pi\omega/2a}d_K^{(2)\dagger} + e^{-\pi\omega/2a}d_{-K}^{(1)}] \qquad (5)$$

So

$$_M\langle 0|b_K^{(1)\dagger}b_K^{(1)}|0\rangle_M = \frac{1}{e^{2\pi\omega/a} - 1} \qquad (6)$$

This means that a Minkowski vacuum state is a thermal equilibrium state for a Rindler observer uniformly accelerated in the Rindler region L. For the region R, we can obtain a similar result

$$_M\langle 0|b_K^{(2)\dagger}b_K^{(2)}|0\rangle_M = \frac{1}{e^{2\pi\omega/a} - 1}$$

Now, we are interested in what a uniformly accelerated observer in the Rindler regions R and L will see when there exists a thermal equilibrium

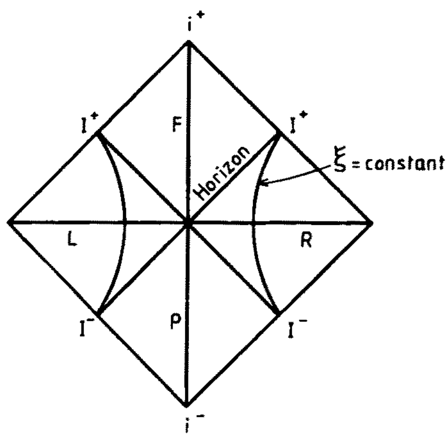

Fig. 1. Conformal diagram of the Rindler system. The regions R, L, F, and P are represented by diamond-shaped regions. ξ = const is the timelike world line of a Rindler observer.

state in the Minkowski spacetime. For simplicity, let us still consider massless scalar particles. With the canonical ensemble, in the Rindler region L, we have

$$\begin{aligned}\langle b_K^{(1)\dagger} b_K^{(1)}\rangle_{M,\beta} &= \mathrm{tr}(e^{-\beta H} b_K^{(1)\dagger} b_K^{(1)})/\mathrm{tr}(e^{-\beta H}) \\ &= e^{\beta F} \sum_i e^{-\beta E_i}[2\ \mathrm{sh}(\pi\omega/a)]^{-1} \\ &\quad \cdot [e^{\pi\omega/a}{}_M\langle\psi_i | N_K^{(2)} | \psi_i\rangle_M + e^{-\pi\omega/a}{}_M\langle\psi_i | (1 + N_{-K}^{(1)}) | \psi_i\rangle_M] \end{aligned} \tag{7}$$

where $N_{-K}^{(1)} = d_{-K}^{(1)\dagger} d_{-K}^{(1)}$, $N_K^{(2)} = d_K^{(2)\dagger} d_K^{(2)}$, and the partition function $e^{-\beta F} = \mathrm{tr}(e^{-\beta H})$. Here $|\psi_i\rangle_M$ is a pure state in the Minkowski spacetime, containing particles $N_K^{(1)}$ and $N_K^{(2)}$ and being an eigenstate of the total Hamiltonian H with energy eigenvalue E_i.

Since the state is in thermal equilibrium for Minkowski (inertial) observers, we have

$$n_{-K}^{(1)} = e^{\beta F} \sum_i e^{-\beta E_i}{}_M\langle\psi_i | N_{-K}^{(1)} | \psi_i\rangle_M = \frac{1}{e^{\beta\omega} - 1} \tag{8}$$

$$n_K^{(2)} = e^{\beta F} \sum_i e^{-\beta E_i}{}_M\langle\psi_i | N_K^{(2)} | \psi_i\rangle_M = \frac{1}{e^{\beta\omega} - 1} \tag{9}$$

The normalization is

$$e^{\beta F} \sum_i e^{-\beta E_i}{}_M\langle\psi_i | \psi_i\rangle_M = 1 \tag{10}$$

So, we obtain

$$\langle b_K^{(1)\dagger} b_K^{(1)}\rangle_{M,\beta} = \frac{e^{2\pi\omega/a} + e^{\beta\omega}}{(e^{2\pi\omega/a} - 1)(e^{\beta\omega} - 1)} \tag{11}$$

In the Rindler region R, we obtain similar result,

$$\langle b_K^{(2)\dagger} b_K^{(2)}\rangle_{M,\beta} = \frac{e^{2\pi\omega/a} + e^{\beta\omega}}{(e^{2\pi\omega/a} - 1)(e^{\beta\omega} - 1)} \tag{12}$$

Equations (11) and (12) tell us that a thermal equilibrium state in the Minkowski spacetime will no longer be an ordinary thermal state for the uniformly accelerated Rindler observer, and it no longer satisfies the Planck distribution law. It will be a new quasi-thermal equilibrium state which is time independent and which is charactered by two quasi-temperature parameters.

From (11), we know that the new state will go over to the ordinary Minkowski thermal equilibrium state

$$\langle b_K^{(1)\dagger} b_K^{(1)}\rangle_{M,\beta} \to \frac{1}{e^{\beta\omega} - 1} \tag{13}$$

when the acceleration a of the Rindler observer tends to zero. On the other hand, when the temperature of the thermal state in the Minkowski spacetime goes to zero, i.e.,

$$\beta \rightarrow \infty$$

the uniformly accelerated Rindler observer will see an ordinary Hawking–Unruh effect,

$$\langle b_K^{(1)\dagger} b_K^{(1)} \rangle_{M,\beta} \rightarrow \frac{1}{e^{2\pi\omega/a} - 1} \tag{14}$$

Summarizing, we find a new quasi-thermal equilibrium state which is charactered by two quasi-temperature parameters. One of them is the initial temperature $1/\beta$. Another is the Hawking–Unruh temperature $T = a/2\pi$. We can name the new state a "double temperature state." It should be pointed out that above conclusion is reliable in principle at least when $1/\beta$ is not too high, although we do not consider the reaction of radiation to the curvature of spacetime.

For the Schwarzschild spacetime manifold, we can get a similar conclusion. The Bogolubov transformations of a massless scalar field between the creation and annihilation operators associated with the Kruskal particles and those associated with the Schwarzschild particles are

$$\begin{cases} b_l^{(1)} = [2 \operatorname{sh}(\pi\omega/\kappa)]^{-1/2}[e^{\pi\omega/2\kappa} d_l^{(2)} + e^{-\pi\omega/2\kappa} d_{-l}^{(1)\dagger}] \\ b_l^{(2)} = [2 \operatorname{sh}(\pi\omega/\kappa)]^{-1/2}[e^{\pi\omega/2\kappa} d_l^{(1)} + e^{-\pi\omega/2\kappa} d_{-l}^{(2)\dagger}] \end{cases} \tag{15}$$

where

$$\begin{aligned} d_l^{(1)} |0\rangle_K &= d_l^{(2)} |0\rangle_K = 0 \\ b_l^{(1)} |0\rangle_S &= b_l^{(2)} |0\rangle_S = 0 \\ \omega &= |l| \end{aligned} \tag{16}$$

κ is the surface gravity of the Schwarzschild black hole. $|0\rangle_K$ and $|0\rangle_S$ are the Kruskal vacuum state and the Schwarzschild vacuum state, respectively. Here, the Kruskal spacetime and the Schwarzschild spacetime are similar to the Minkowski spacetime and the Rindler spacetime, respectively.

It is well known that a Kruskal vaccuum state is a thermal state in the Schwarzschild spacetime

$${}_K\langle 0 | b_l^{(1)\dagger} b_l^{(1)} | 0 \rangle_K = {}_K\langle 0 | b_l^{(2)\dagger} b_l^{(2)} | 0 \rangle_K = \frac{1}{e^{2\pi\omega/\kappa} - 1} \tag{17}$$

where $T = \kappa/2\pi$ is the temperature of the black hole. Similar to equations (11) and (12), we can prove that an ordinary thermal equilibrium state whose temperature is $1/\beta$ in the Kruskal spacetime will be a "double temperature state" in the Schwarzschild spacetime. We have

$$\langle b_l^{(1)\dagger} b_l^{(1)} \rangle_{K,\beta} = \langle b_l^{(2)\dagger} b_l^{(2)} \rangle_{K,\beta} = \frac{e^{2\pi\omega/\kappa} + e^{\beta\omega}}{(e^{2\pi\omega/\kappa} - 1)(e^{\beta\omega} - 1)} \tag{18}$$

Here, similar to the Minkowski case, we do not consider the reaction of radiation to the curvature of spacetime. We believe that the above conclusion is reliable in principle, at least when $1/\beta$ is not too high.

The "double temperature state" phenomenon may be quite general in quantum field theory in curved spacetime. It is a kind of new quasi-thermal equilibrium state. Their energy spectra are not Planck spectra. They are characterized by two quasi-temperature parameters, unlike ordinary thermal equilibrium states, which are characterized by a single parameter called the temperature. The validity of the conclusion that the usual thermal state in Minkowski spacetime (or Kruskal spacetime) is no longer a thermal state for a Rindler (respectively Schwarzschild) observer can also be seen from the fact that the Kubo–Martin–Schwinger (KMS) (Kadanoff and Baym, 1962; Haag *et al.*, 1967) condition which characterizes a thermal state is not preserved under the passage from the Minkowski to the Rindler frame, or from the Kruskal to the Schwarzschild frame.

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